

High temperature meson propagators with domain-wall quarks.*

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We study the chiral properties of domain-wall quarks at high temperatures on an ensemble of quenched configurations. Low lying eigenmodes of the Dirac operator are calculated and used to check the extent to which the Atiyah-Singer index theorem is obeyed on lattices with finite N_5 . We calculate the connected and disconnected screening propagators for the lowest mass scalar and pseudoscalar mesons in the sectors of different topological charge and note that they behave as expected. Separating out the would-be zero eigenmodes enables us to accurately estimate the disconnected propagators with far less effort than would be needed otherwise.

1. Introduction

We use Shamir's formulation [1] of lattice domain-wall fermions, which have exact chiral flavour symmetry for infinite 5th dimension (N_5), to study the chiral properties of quarks in high temperature QCD and their relation to topology.

The chirally symmetric Dirac operator should have a zero mode associated with each instanton, and obey the Atiyah-Singer index theorem. In addition, these modes yield the disconnected contributions which distinguish the π from the η' screening propagators and the $\sigma(f_0)$ from the $\delta(a_0)$ propagators, and give related contributions to the connected propagators. Since we use quenched configurations, we cannot determine whether the anomalous $U(1)$ axial symmetry is restored in the high temperature phase of the massless $N_f = 2$ theory, but we can check relations required for the $U(1)$ axial symmetry to remain broken.

We measured eigenmodes of the domain-wall Dirac operator as a function of N_5 on a set of quenched configurations in the high temperature phase. We check the Atiyah-Singer theorem for $N_5 = 10$ and calculate connected and disconnected scalar and pseudoscalar screening propagators. Early results were presented at Lattice'98 [2]. Section 2 describes our analysis and tests of the index theorem. The meson screening propagators are discussed in section 3. Section 4 gives

our discussions and conclusions.

2. Domain-wall quarks at high temperatures

For our study of domain-wall quarks at high temperatures we use $16^3 \times 8$ quenched configurations at $\beta = 6.2$ (170), $\beta = 6.1$ (100) and $\beta = 6.0$ (100). (At $N_t = 8$, $\beta_c \approx 6.0$.) On each configuration, we estimated the topological charge by the cooling method.

We studied eigenvalue trajectories of the hermitian Wilson Dirac operator $\gamma_5 D_W$ as the bare mass was varied. These vanish at would-be zero modes associated with instantons [3]. Based on these studies we chose the Shamir domain-wall mass parameter $M = 1.7$ for all 3 β values.

We calculated the lowest 2 eigenmodes of the hermitian domain-wall Dirac operator $\gamma_5 \mathcal{R} D_{dw}$ for all $\beta = 6.2$ configurations with non-trivial topology and for all $\beta = 6.1$ and $\beta = 6.0$ configurations, for $N_5 = 4, 6, 8$ and 10. (A third $N_5 = 10$ eigenmode was calculated for each $\beta = 6.0$ configuration with topological charge ± 3 .)

For $\beta = 6.2$, there is a clear separation between would-be zero modes whose eigenvalues decrease exponentially with N_5 and non-zero modes which rapidly approach a constant. By $\beta = 6.0$ the situation is less clear. For $\beta = 6.2$, $N_5 = 10$ appears adequate.

We calculate the chiral condensates $\langle \bar{q}q \rangle$ and $\langle \bar{q}\gamma_5 q \rangle$ for the case $N_5 = 10$, using an eigenmode enhanced stochastic estimator. For each configu-

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ration, exact chiral symmetry requires $\langle \bar{q}\gamma_5 q \rangle$ to obey the Atiyah-Singer index theorem:

$$m \sum_x \langle \bar{q}(x) \gamma_5 q(x) \rangle_U = Q_U. \quad (1)$$

As figure 1 shows, the index theorem at $\beta = 6.2$ is well approximated down to masses comparable to the eigenvalue of $\gamma_5 \mathcal{R}D_{dw}$ at $m = 0$ of smallest magnitude.

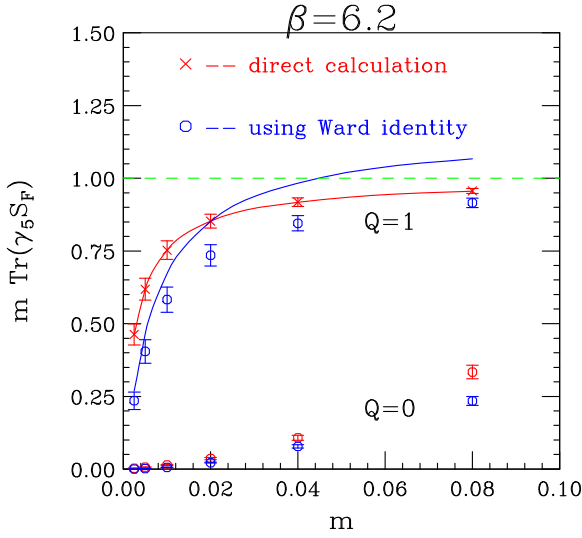


Figure 1. $m\text{Tr}(\gamma_5 S_F)$ as a function of mass for $\beta = 6.2$. The solid lines for $Q = 1$ are the contributions from the lowest eigenmode.

3. Meson screening propagators.

Screening propagators, i.e. propagators for spatial separations, describe the excitations of hadronic matter and the quark-gluon plasma at finite temperature. We measure the connected and disconnected parts of these propagators for scalar and pseudoscalar mesons with zero momenta transverse to their separation (Z).

The connected parts of these propagators are measured with a noisy point source on 1 z slice for

$Q = 0$ and on each z slice for $Q \neq 0$. The disconnected parts of these propagators are measured separating out the contributions of the eigenmodes (projected on to the domain walls) and approximating the remainder of the required traces with a stochastic estimator. For a discussion of stochastic estimators for staggered quarks see[4].

The $\beta = 6.2$ disconnected propagators behave as expected. For $Q = 1$, the contribution is sizeable and increases as quark mass decreases (it should diverge as $1/m^2$ if chiral symmetry were exact), as shown in figure 2. For $Q = 0$ the con-

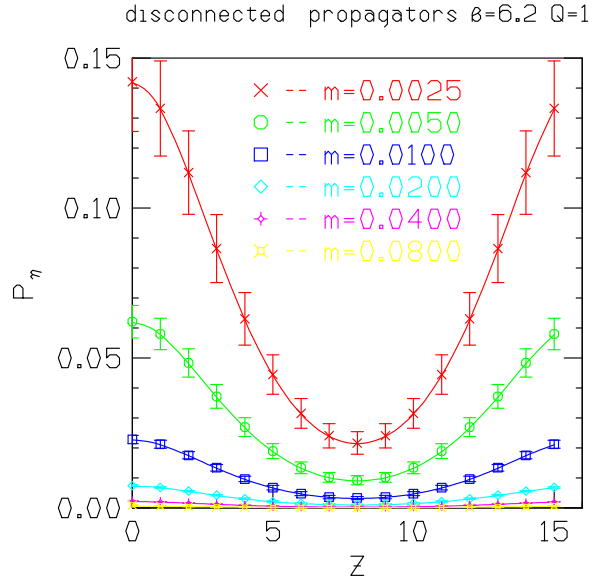


Figure 2. The disconnected contribution to the η' propagator at $\beta = 6.2$. The corresponding graph for the σ is virtually identical.

tribution is extremely small compared with the connected contribution (on the scale of figure 2 all points would lie on the horizontal axis). This is consistent with the expectation that the $Q = 0$ contribution vanishes in the chiral limit.

For the connected propagators, we consider the combinations $\frac{1}{2}(P_\pi \pm P_\delta)$ (Note: the connected part of the η' propagator is the π propagator and

the connected part of the σ propagator is the δ propagator). For $Q = 0$, flavour chiral symmetry restoration and the absence of a disconnected contribution would make $\frac{1}{2}(P_\pi + P_\delta)$ vanish in the chiral limit. The observed value is very small. The other combination, $\frac{1}{2}(P_\pi - P_\delta)$, is non-vanishing and has a finite chiral limit, as expected.

In the $Q = 1$ sector, $\frac{1}{2}(P_\pi + P_\delta)$ should equal the disconnected part of the σ and η' propagators in the chiral limit, for chiral flavour symmetry restoration with broken $U(1)$ axial symmetry to be possible for $N_f = 2$ [5]. $\frac{1}{2}(P_\pi + P_\delta)$ is plotted in figure 3. Figures 3 and 2 are consistent with

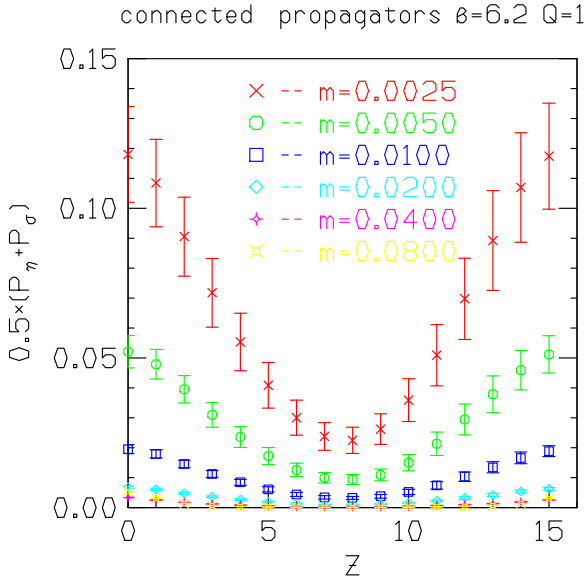


Figure 3. The connected propagator $\frac{1}{2}(P_\pi + P_\delta)$ for $Q = 1$ at $\beta = 6.2$.

this expectation.

4. Discussion and conclusions.

For quenched lattice QCD at $N_t = 8$, domain-wall quarks with $N_5 \geq 10$ are a good approximation to chiral quarks at $\beta = 6.2$ and suggest an approach to chiral symmetry which is exponen-

tial in N_5 . The index theorem is well approximated and the π , σ , η' and δ propagators show the correct behaviour for the restoration of chiral $SU(2) \times SU(2)$ flavour symmetry for $N_f = 2$. The disconnected parts of the σ and η propagators come entirely from the $Q \neq 0$ sector and have the behaviour needed to give a finite contribution when the $N_f = 2$ determinant is included. Isolating the lowest eigenmodes greatly improves our stochastic estimation of traces.

Our preliminary results for $\beta = 6.0$ indicate a more complex behaviour. It is not yet clear whether this indicates that $N_5 = 10$ is too small, or that the condensation of instanton-antiinstanton pairs, as observed by Heller et al. for $N_t = 4$ using Ginsparg-Wilson fermions [6], is playing an important role.

A dynamical simulation is needed to determine if, for $N_f = 2$, the $U(1)$ axial symmetry remains broken above the transition. Such simulations are being performed by the Columbia group at $N_t = 4$ [7], and preliminary indications are that it does remain broken. These pioneering results are limited by the strong couplings dictated by $N_t = 4$, and use of heavier quarks than would be desirable. To push closer to the chiral limit probably requires isolating the lowest eigenmode(s) in the simulations.

These calculations were performed on the C90 and J90's at NERSC.

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connected propagators $\beta=6.2$ $Q=0$

